⌈ (BC Only)

22. Integrate by parts. Let $u = x$ and $dv = e^{2x}dx$; $du = dx$; $v = \frac{1}{2}e^{2x}$. So

$\int xe^{2x}dx = \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x}$. Evaluating the definite integral gives

$$\int_1^2 xe^{2x}dx = \left(\frac{x}{2}e^{2x} - \frac{1}{4}e^{2x}\right)\bigg|_1^2 = \left(e^4 - \frac{1}{4}e^4\right) - \left(\frac{e^2}{2} - \frac{e^2}{4}\right) = \frac{3e^4}{4} - \frac{e^2}{4}.$$

23. Integrate by parts. Let $u = \ln x$ and $dv = dx$; $du = \frac{1}{x}dx$; $v = x$. So

$\int \ln x dx = x\ln x - \int x \cdot \frac{1}{x}dx = x\ln x - x$. Evaluating the definite integral gives

$$\int_1^{\sqrt{e}} \ln x dx = x\ln x - x\Big|_1^{\sqrt{e}} = \left(\sqrt{e}\cdot\frac{1}{2} - \sqrt{e}\right) - (0-1) = 1 - \frac{\sqrt{e}}{2}.$$

24. Use partial fractions: $\frac{1}{t^2+2t} = \frac{1}{t(t+2)} = \frac{A}{t} + \frac{B}{t+2}$. Multiplying gives

$1 = A(t+2) + Bt \Rightarrow B = -\frac{1}{2}$ and $A = \frac{1}{2}$. So

$$\int_1^2 \frac{1}{t^2+2t}dt = \frac{1}{2}\int_1^2 \frac{1}{t}dt - \frac{1}{2}\int_1^2 \frac{1}{t+2}dt = \frac{1}{2}\left[\ln t\Big|_1^2 - \ln(t+2)\Big|_1^2\right] =$$
$$\frac{1}{2}\left[\ln 2 - (\ln 4 - \ln 3)\right] = \frac{1}{2}\ln\left(\frac{3}{2}\right).$$

25. This is an improper integral, since the integrand has a vertical asymptote at $x = 3$.

By definition, $\int_3^4 (x-3)^{-\frac{1}{2}}dx = \lim_{a\to 3}\int_a^4 (x-3)^{-\frac{1}{2}}dx = \lim_{a\to 3} 2(x-3)^{\frac{1}{2}}\Big|_a^4 = 2$.

26. By definition, $\int_{-\infty}^1 e^{2x}dx = \lim_{a\to-\infty}\int_a^1 e^{2x}dx = \lim_{a\to-\infty}\frac{1}{2}e^{2x}\Big|_a^1 = \frac{1}{2}e^2$.

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